Lattice kinetic formulation for ferrofluid dynamics

Paul J. Dellar
OCIAM, Mathematical Institute, University of Oxford
24-29 St Giles', Oxford, OX1 3LB, United Kingdom
pdellar@na-net.ornl.gov

Ferrofluids are suspensions of magnetic particles (perhaps nanometre sized) in an insulating liquid such as toluene. The magnetic moments of the particles allow the ferrofluid to be subjected to body forces by imposing a suitable magnetic field. Since no currents can flow in the insulating base fluid, the body forces are purely due to the magnetic interacting with the dipole moments, rather than with electric currents as in ordinar magnetohydrodynamics. Ferrofluids are therefore not subject to the energy losses from resistive heating that limit attempts to manipulate ordinary electrically conducting fluids with magnetic fields.

The author's previous lattice kinetic formulation for magnetohydrodynamics [1] uses a vector-valued distribution function to represent the magnetic field. This enables the evolution equation for the magnetic field to be expressed in conservation form with the diverence *antisymmetric* flux tensor, equivalent to the curl of the electric field as required by Maxwell's equations. The corresponding tensor computed from a scalar distribution function, analogous to the momentum flux or stress in a lattice Boltzmann formulation of ordinary hydrodynamics, is forced to be a symmetric tensor. The magnetic field thus cannot evolve correctly if represented using a scalar distribution function. The vector-valued formulation has been shown to be competitive with other numerical methods, spectral and upwind projection, for computing the reconnection of magnetic islands.

The formulation is now extended to ferrofluids, using a vector-valued distribution function to represent the magnetisation. Another novel feature of ferrofluids is that the particles may spin relative their surrounding fluid under the influence of magnetic torques. Our treatment is restricted to short spin relaxation times, in order to achieve the symmetric total stress tensor necessary for a kinetic theory formulation of the hydrodynamics, but allows the finite spin viscosity that is required to explain effects like magnetoviscosity, the apparent increase in viscosity deduced from Poiseuille flow of a ferrofluid in a magnetic field.

[1] P. J. Dellar, *Lattice kinetic schemes for magnetohydrodynamics*, J. Comput. Phys. **179**, 95 (2002).