A new approach for establishing lattice-Boltzmann models from the continuous Boltzmann equation

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The continuous Boltzmann equation (BE) can, now, be envisaged as a consistent framework for the systematic derivation of discrete velocity models, after He & Luo, [1], have directly derived the lattice Boltzmann equation (LBE) from it by discretization in both time and velocity space. Although limited to a single relaxation time (SRT), the use of a BGK model for the collision term enabled the construction of implicit numerical schemes, [2], or LBE modified explicit numerical schemes, [3], with third order time step, δ, errors. On the other hand, it is well known that the correct description of fluids and fluid flow requires the multiple relaxation time models (MRT), firstly introduced in the LBE framework by D'Humières, [4]. Nevertheless, errors $O(\delta^3)$ cannot be achieved in the presently known MRT models. In spite of the fact that, in athermal models, this truncation error can be totally absorbed into the physical viscous term, in thermal models, errors $O(\delta^2)$ seriously affects the viscous heat dissipation term. In present work, a new procedure for systematically deriving lattice-Boltzmann models from the continuous Boltzmann equation is proposed, combining these two main features: multiple relaxation-time and $O(\delta^3)$ time step errors. The collision term $\mathcal{L}(f^{neq})$ in the linearized Boltzmann equation is modeled by expanding the distribution function f in Hermite tensors $\mathbf{\Psi}_\theta$, which forms an orthogonal basis in the D-dimensional velocity space $\boldsymbol{C}^D$. Considering that each term $\mathcal{L}(\mathbf{\Psi}_\theta)$ is, itself, an element of $\boldsymbol{C}^D$, this term is expanded as a linear combination of the same order-θ Hermite tensors through 2θ-order relaxation tensors. Isotropy properties are used to reduce these tensors. The infinite series $\mathcal{L}(f^{neq})$ is not truncated. Instead, after a chosen tensor order n, the relaxation tensors are diagonalised resulting in an absorption term $\lambda_n f^{neq}$. For each n, equilibrium distribution is taken as the n^{th}-order Hermite expansion of to the Maxwell-Boltzmann (MB) equilibrium distribution. This is an additional distinguishing feature with respect to the previous models in which either the equilibrium distribution is considered as a small macroscopic speed expansion of the MB distribution, [1], or constructed in order to retrieve the correct macroscopic hydrodynamics equations (MRT-based models), [5]. The derived kinetic model of the continuous Boltzmann equation is then discretized. Sample lattice Boltzmann equations are derived from the gaussian-hermite quadrature of the n-th order approximation to the MB distribution. In thermal problems, a modified procedure is proposed for taking the temperature dependence of lattice velocities into account.

References

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