Energy-conserving Lattice Bolzmann Thermal Model in 2 dimensions

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Over the last decade, it has been demonstrated that Lattice Boltzmann Method (LBM) is an effective numerical method to simulate a wide variety of isothermal fluid flows [1].

In the case of thermal fluid flows, LBM with a multi-speed approach under a single relaxation time BGK approximation, suffers from numerical instabilities [2]. To avoid these instabilities, the passive scalar approach [3] or introduction of a separate thermal distribution [4] can be used. Vahala *et al.* [5] have proposed a multi-speed model with a higher-order-isotropy velocity grid and multiple relaxation times to stabilize the numerical scheme and to have a variable Prandtl number. In [5] a model was suggested based on Gauss-Hermite quadrature that is a straightforward extension of the 'a priori' derivation of the Lattice Boltzmann equation by He and Luo [6]. To include thermal effects, heat conduction and viscous heat dissipation, the quadrature must be used to evalute the moments of $f^{(eq)}$ to the eighth order. So the Lattice Boltzmann Thermal Model needs 5 discrete velocities in one dimension and 25 discrete velocities in two dimensions.

In the present work, as in [4] a second-order strategy is adopted to integrate the Boltzmann equation, in order to avoid, after the Chapman-Enskog expansion, the inconsistancy between the viscosity in the momentum and energy equation. There are two difficulties in implementing this model. The discrete velocities are temperature dependent and the second velocity is not exactly equal to twice the first velocity. To treat the temperature dependence, the phase space is discretised with an average temperature and the weights of the quadrature are recalculated at each time step with the new temperature field. In other words, the temperature dependance is reported into the weights. To treat the problem of "non-double" velocities there are two possibilities : linear interpolation of f in real space or an extrapolation in velocity space assuming f has the form of $f^{(eq)}$. These two possibilities have been evaluated numerically and their relative merits are presented. Application of the numerical scheme is illustrated by two *2D* examples.

References

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