Asymptotic-Preserving Schemes for Unsteady Flow Simulations

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The main purpose of our contribution is concerned with extension of relaxation approximation to unsteady flows at all speeds. Given a nonlinear kinetic equation of Boltzmann type, we construct a system of conservation laws with relaxation term that yields in the limit for small Knudsen and Mach numbers an approximation of fluid dynamic equations. To solve numerically the relaxation system we introduce an asymptotically preserving space and time discretizations. Advantages of relaxation method are the semilinear construction of the approximating system and a special time implicit-explicit splitting for the relaxation term. The first advantage allows to compute the numerical solution of the system without introducing Riemann solvers and the second avoid the use of nonlinear system of algebraic equations solver. To be more precise our talk deal with the two following features of relaxation methods:

1. Reconstruction of high order relaxation method based on combining higher order discretization in space and higher order implicit-explicit Runge Kutta methods for the time integration. At the limit these schemes are shown to be TVD.

2. Application of the method to compressible and incompressible flows. For instance, inviscid Euler system in gas dynamic and viscous incompressible Navier-Stokes are studied with more details.

The strong relationship between the nature of nonlinear kinetic equations and the choice of the most appropriate relaxation method working uniformly in the fluid dynamic limits is also highlighted for some of these results. These methods preserve the main physical properties: positivity, conservation of mass, momentum and energy.

Finally, several numerical tests on known benchmark problems are presented in order to show the high resolution of the method in the presence of sharp gradients and high Reynolds numbers.

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