

The Cumulant Method: Theory, Implementation and Numerics

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The talk will present a new method for approximate numerical solution of the Boltzmann equation. The cumulant method is a mesoscopic method for approximate solution of the Boltzmann equation similar to moment methods.

The presentation will start with a short review of the various moment methods and discuss their properties, advantages and drawbacks. The second part will introduce in detail the cumulant method of reducing the Boltzmann equation to a set of partial differential equations. Compared to moment methods, these equations have a particularly simple structure and constitute a true hierarchy of equations, that is, the closure of low order equations is not dependent on the order of the method. This is not the case for most known moment systems. For the case of a 2D Maxwell gas mixture, production terms can be calculated analytically and we demonstrate a method for derivation of the production terms in full non-linear form. It appears that for the first three cumulant-orders the production terms are linear in the cumulants. Next we discuss a linearized variant for states close to a local equilibrium. The eigensystem of the linearized collision terms allows to discuss the relaxation behaviour of the gas and enables us to identify the first eigenvariables of the production terms as well-known thermodynamic quantities. The third part will present first numerical results on the domain of hyperbolicity, discuss some possible models for the treatment of various boundary conditions and present exemplary numerical results for simulations of the Bobylev/Krook/Wu problem and Poiseuille and Couette flow situations.